Impedance Correction of Wave Filters

Development of Impedance Requirements

By E. B. PAYNE

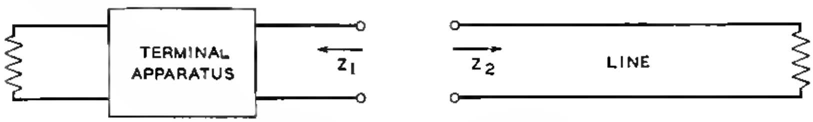
The present importance of wave filter impedance correction arises chiefly from its relation to crosstalk in carrier systems. Briefly, it appears that line transpositions, an effective remedy for many types of crosstalk, are less satisfactory when directed against the so-called "reflected near-end crosstalk" and "reflected far-end crosstalk" produced when waves reflected from the junctions between lines and repeater equipment of carrier systems induce currents in neighboring systems. The expense of the elaborate transposition scheme necessary for a substantial reduction in these types of crosstalk makes it desirable to diminish the amplitude of the reflected wave as far as possible by the improvement of the impedance match between lines and repeaters. A detailed study shows that this is most conveniently done by terminating the filters in the repeater by sections whose image impedances at one end match the main body of the filter, while at the other they approximate constant resistances, matching the terminal impedances.

The development of appropriate filter terminating sections has passed through a number of stages. The earliest filters gave reflection coefficients as great as 50% to 60% in the useful transmission band. The invention of "m-derived" and "x-terminated" filters, plus a number of more or less empirical schemes, made it possible to obtain reflection coefficients ranging from 10% to 15% in the useful band. Recent progress has resulted chiefly from the development of a series of sections, the simplest of which is equivalent to the m-derived type, while the others, of progressively increasing complexity, give progressively better approximations to the ideal characteristic. The use of the more complicated sections has made it possible to reduce filter reflection coefficients to the order of 2%, or even less. At present the chief limitation appears to be the difficulty of manufacturing filters with sufficient precision to allow the theoretical characteristics to be realized. The paper is illustrated by figures showing the various stages of this progress as they are exemplified in actual designs.

THE rapid increase in the demand for long distance or toll telephone service in recent years led to the introduction, about 1920, of carrier systems as a means of securing more intensive use from long telephone lines. The growth of these circuits has resulted, still more recently, in the multiplication of the number of carrier systems in use and in the close association of several similar or different carrier systems on a single pole-line. This development raised a number of totally new engineering problems and demanded careful reconsideration of many other questions of comparatively small importance in earlier systems.

Among the factors thus brought into prominence by carrier system development, the chief, for the purpose of this paper, is the impedance mismatch between telephone lines and repeaters or terminal apparatus. The components of a complete transmission system, such as the line

itself, various transmission networks, amplifiers, modulators, electro-acoustic apparatus, etc. are quite dissimilar physically and as we might naturally expect, these physical differences manifest themselves in many instances as pronounced dissimilarities in the forms of the impedance-frequency characteristics. For example, the characteristic impedance of a uniform line varies smoothly with frequency, but that of a wave filter changes abruptly as we go from the transmitting to the attenuating range. In spite of the possibility of changing the general impedance level by the insertion of a transformer such inherent "incompatibilities of temperament" between the characteristic impedances of the various components of the telephone circuit must lead normally to impedances which resemble each other only in narrow frequency bands and which may differ widely over large and important portions of the frequency spectrum. In default of some method of extending the range of similarity, most long telephone circuits will exhibit wide impedance mismatches or irregularities at numerous junction points.



$$\text{REFLECTION COEFFICIENT} = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$

Fig. 1—Junction of line and terminal apparatus illustrating impedances which determine the reflection coefficient.

In voice frequency circuits or in carrier circuits which are not in close physical association, impedance irregularities are of importance only insofar as they affect transmission efficiency.* In addition to modifying the current which proceeds onward toward the receiving device, however, an impedance difference at any junction produces a reflected wave which retraverses the circuit toward the sending end. A convenient measure of this second effect is found in the "reflection coefficient" which may be defined as the vector difference of the two impedances looking both ways from any junction divided by their vector sum (see Fig. 1) and is equal both in magnitude and phase to the ratio between the reflected wave and the wave originally propagated.

The effect of reflection of considerable magnitude on transmission is slight. Indeed relatively large reflection may actually improve the transmission characteristic of certain circuits. In voice frequency circuits and in carrier circuits which are not operated over lines in close

* In two wire repeatered circuits reflection causes echoes which are one of the limiting factors of such circuits. These circuits are, however, outside the scope of this paper.

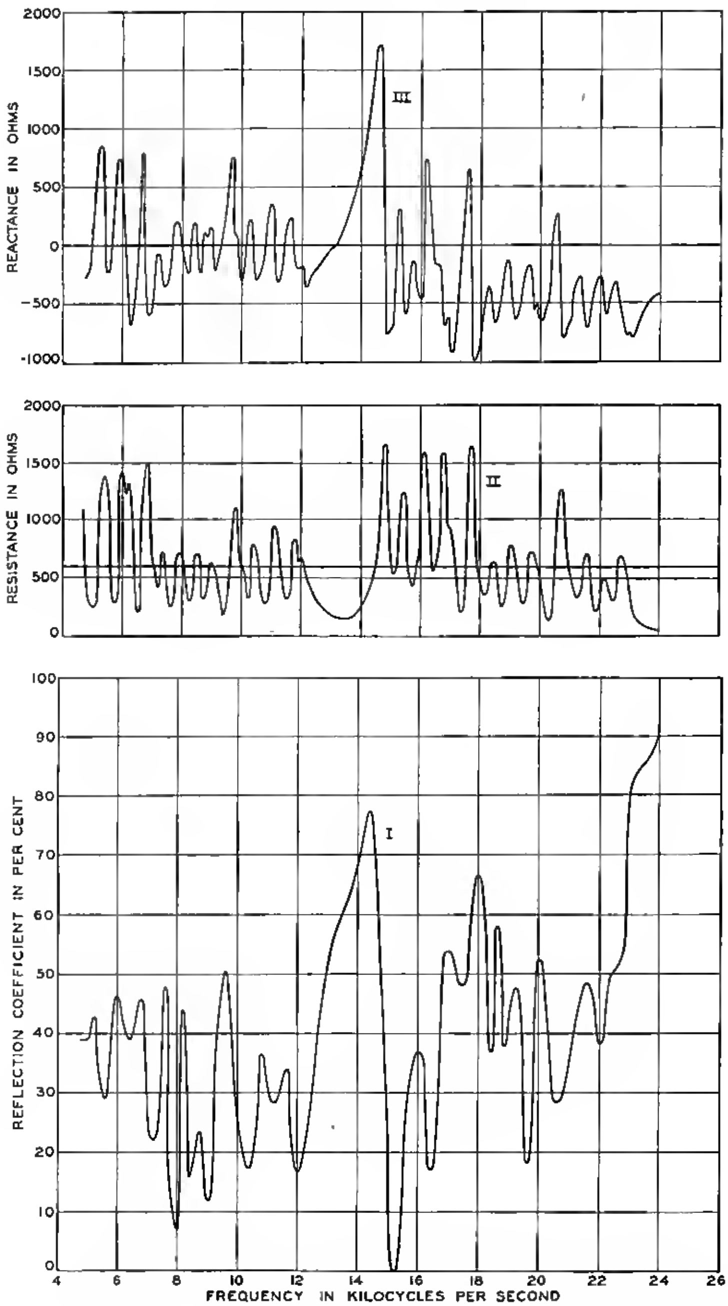


FIG. 2—Impedance and reflection coefficient of an early carrier telephone system.

physical association, transmission is the only consideration. Consequently relatively large reflection coefficients are not objectionable in such circuits. Curve I of Fig. 2 shows the reflection coefficient of an early carrier system which was not intended to work with other systems of the same type. Large as these reflections appear relative to present day standards they did not seriously impair the transmission of the system. It is also true that appreciably better results could not have been obtained with the design technique available when the filters for the system were developed.

As the increase in the demand for long distance traffic made it necessary to associate systems on the same pole line the situation became radically different through the introduction of a new factor crosstalk between systems. Crosstalk between systems at carrier frequencies is inherently large and the methods of reducing it expensive. The reflections due to the mismatching in systems increase the crosstalk between them by introducing a type of interference known as "reflected near-end crosstalk." This type of crosstalk can be made negligible only by making the impedance mismatching in the two systems very small. Since "near-end crosstalk" contributes heavily to the cost of the arrangements for reducing crosstalk between carrier systems the substantial elimination of impedance irregularities between lines and the filters and associated apparatus composing the terminals of systems becomes of great economic importance.

As this need for reducing and ultimately eliminating these irregularities appeared a series of improvements in design technique have been developed, each better than its predecessor, which have culminated in a technique which appears to be adequate for the purpose. It differs in many essential features from the others and leads to a new type of filter section which is not of the standard recurrent ladder type. It is the purpose of this paper to give some idea of the relation of crosstalk to impedance mismatching, show how the successive stages of the filter development have grown out of the system requirements and finally to present an outline of the final technique. The accompanying paper. "A Method of Impedance Correction." by H. W. Bode gives this technique in detail.

Impedance Irregularities and Crosstalk

The ultimate relation between reflection and crosstalk between two lines which are associated with a number of others on telephone poles is extremely complicated. An idea of the principles which underlie the relationship may be obtained by considering only two of the circuits and assuming that the others have been temporarily removed.* If these

* Since these two circuits consist of two pairs of wires, there is a potential phantom

two telephone lines parallel each other, as in Fig. 3, they will be electrically coupled through mutual inductance and capacity. Currents flowing in one will consequently produce crosstalk currents in the other. When the subscriber at the west end of the line (A) is talking, waves initiated by his voice will cross from line (A) to line (B) at adjacent points along the entire length of the lines.

Crosstalk entering line (B) at a typical point may traverse four chief paths. It may (1) flow directly back to the west end, (2) flow onward to B'' and be reflected back to the west end, (3) flow directly onward to the east terminal, and (4) flow backward to B' and thence, by reflection, to B''. There are of course an infinite number of other paths involving multiple reflections but the reduction in amplitude caused by successive

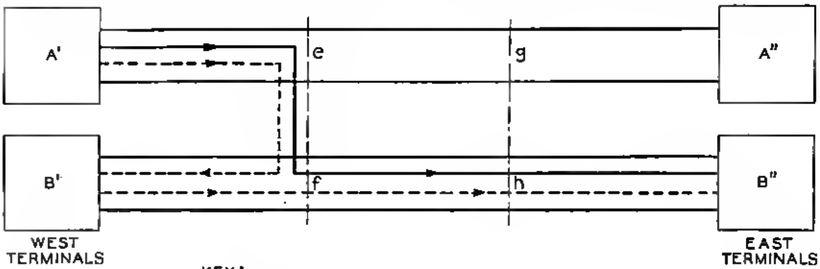


Fig. 3—Diagram illustrating relation between impedance mis-matches and crosstalk in carrier systems.

reflection and line attenuation makes these negligible in comparison with the others. The first two of these four possibilities of crosstalk production can be eliminated immediately. Modern carrier systems are so designed that conversations going in one direction are carried by one band of frequencies and those travelling in the opposite direction by a different band. Currents entering the west terminal, whether they follow a direct path such as $AefB'$, or are first reflected at B'', making the typical path $A'efB''B'$, are therefore eliminated by the filters in the terminal office. Crosstalk currents of the third type ("far end crosstalk") following the typical path $A'efB''$ cannot be eliminated since they fall within the frequency band used by the subscriber at B'' for listening. We may observe, however, that these currents traverse the same length of line in travelling from A' to B'' no matter what the point (such as ef in the diagram) at which they cross from one line to the other. Since both lines are alike crosstalk currents will be attenuated and shifted in phase by the attenuation and phase shift of a single

circuit which, as far as crosstalk is concerned, constitutes a third circuit. The effect of this circuit on the crosstalk of the other two is by no means negligible but consideration of it is omitted herein in order to simplify the presentation of other important relations fundamental to crosstalk.

full length line. Moreover the components of this type of crosstalk due to magnetic and capacitative coupling are nearly out of phase and so one tends to neutralize the other. As a consequence of this equal effect of the line characteristic on all the components which reach the receiver at B'' the resultant crosstalk can theoretically be eliminated at all frequencies when only two circuits are present by a single transposition (crossing the wires) in the center of either line.

Crosstalk currents of the fourth type, "reflected near-end crosstalk" following such paths as $A'efB'B''$ and $AghB'B''$, cannot, however, be disposed of so easily. The length of line traversed by the component currents which make up the resultant crosstalk depends upon the point at which they cross from one line to the other, and they will therefore be affected in various fashions by the line attenuation and phase shift. The transposition scheme required to eliminate crosstalk resulting from these currents will consequently depend, at any frequency, upon the length of the line and upon its phase and attenuation characteristics at that frequency. Complete elimination of crosstalk of the fourth type cannot be secured, even for two circuits over a finite frequency band, from a finite number of transpositions.

When other lines are adjacent to the two we have considered the problem of reducing "far-end" and "near-end" crosstalk by transpositions is still more complicated. With a number of lines it is no longer even theoretically possible to eliminate far-end crosstalk by a single transposition. It is, in general true, however, that the cost of a transposition scheme adequate for far-end crosstalk is still much less than that of the elaborate system of transposition required to reduce near-end crosstalk to tolerable values. From an economic standpoint therefore, the cost of transpositions required for near-end crosstalk is usually the main feature to be considered.

Impedance Correction an Economic Means of Controlling Crosstalk

Another method of reducing this near-end crosstalk, and one which experience has shown to be much cheaper than an elaborate transposition scheme is found in the reduction of the reflection coefficient between the line and the repeaters. Obviously the magnitude of the reflected near-end crosstalk depends upon the amount of the impedance mismatch at the junction between the line and the terminal equipment (e.g. at B' in Fig. 3). It can be made as small as we please, even with a very simple transposition scheme, if the reflection coefficient at line-repeater junctions can be sufficiently reduced. No serious mismatches would occur if the impedances of repeaters and terminal equipment were that of the modulators or amplifiers, since at carrier frequencies

the impedances of modulators, amplifiers and telephone lines approximate constant resistances. The interposition of filters between lines and modulating or amplifying apparatus, however, normally produces large reflection coefficients. Since the filters in addition to being the apparatus immediately responsible for mismatching, are also inexpensive and easily controlled in comparison with the line, they furnish the most promising field for the reduction of reflection coefficients.

Relation between Actual and Image Impedances

The reflection coefficient which determines the amount of crosstalk exhibited by the system involves directly only the line impedance and the *actual* impedance characteristic of the filter system. In order to understand the peculiarities of the actual impedance of a filter, however, it is necessary to give prior consideration to its *characteristic*, or *image* impedances. The image impedances of any transmitting device are defined as the impedances with which the device must be terminated at both ends if the impedances looking both ways at each pair of terminals are to be matched. In other words, they are the impedances with which the structure must be terminated if no reflections are to occur. Filter sections of different physical configuration and with different attenuation characteristics often have the same image impedance characteristics. Practical filter designs are therefore usually composite structures containing several different types of sections. Internal reflections are avoided by so choosing the arrangement of the section that the image impedance characteristics at all section junctions are matched. Under these circumstances the image impedance characteristics of the complete structure are the same as those of its terminating sections. The image impedance characteristic of typical low pass filter sections is shown in Fig. 4-*A*. A corresponding curve for band pass filters is given in Fig. 4-*B*. The image impedance characteristics are given only for the transmitting bands of the filters since, as previously indicated, the filters themselves suppress crosstalk in the attenuating regions making it unnecessary to control impedances outside the transmitting range. The associated equipment, such as lines and modulators, with which the filters are terminated, are approximately constant pure resistances, and may be represented in the transmitting range by the block type characteristics drawn over the curves.

The relation between the image impedance properties of filters and the actual impedance presented by a repeater or terminal to the line can be understood from the simplified circuit diagram of a typical carrier terminal (Fig. 5). Upon examining the figure we note that there are a number of junctions at which rounded filter image im-

pedance characteristics such as those shown in Fig. 4-*A* or Fig. 4-*B* face the block type image impedances of the same figure. In the system of Fig. 5, for example such junctions occur at *B*, *C*, *D*, *E*, and to some

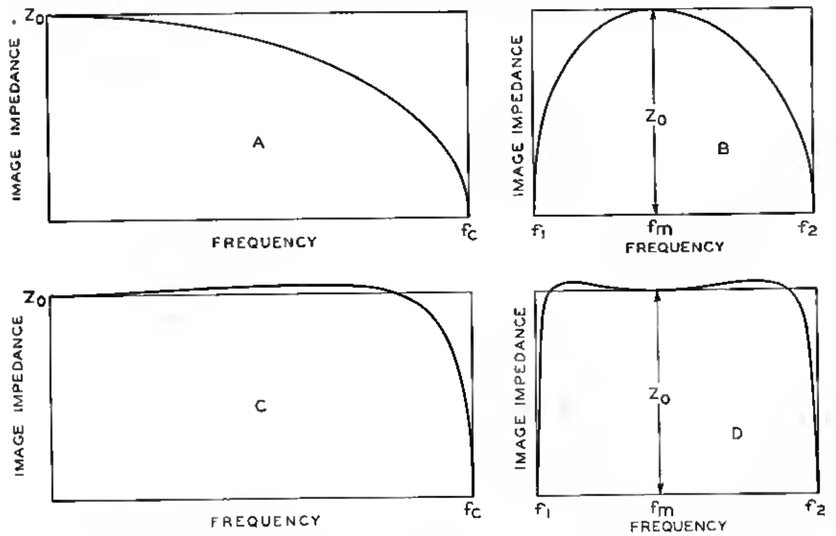


Fig. 4—Image impedances of "constant-*k*" and "m-derived" low-pass and band-pass sections.
Figs. 4-A and 4-B—"constant-*k*" sections.
Figs. 4-C and 4-D—"*m*-derived" sections.

extent at *A*. It is evident, of course, that reflected waves will be produced at these points, and since impedance differences occur at several junctions a wide variety of multiple reflections may exist.

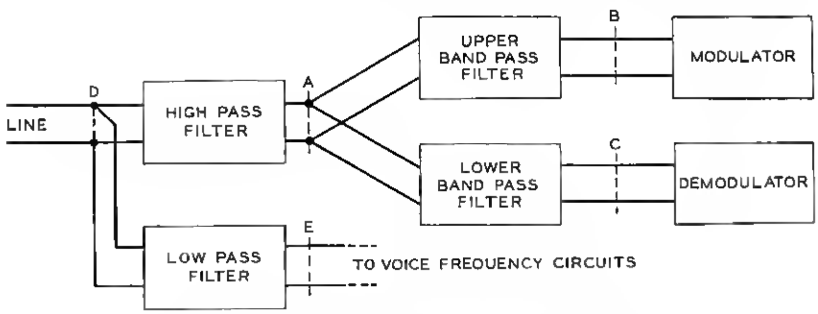


Fig. 5—Simplified circuit diagram of a typical carrier terminal.

Upon reaching the line terminals, all of these reflections combine with the wave originally propagated to determine the actual current entering the structure. The reflected waves are of course diminished in

magnitude in traversing the filters intervening between the line terminals and the junctions at which mismatches occur. The attenuation of most filters within transmitting bands is so small however, that the waves may be of appreciable magnitude even after several reflections. The filter phase shift within these frequency bands, on the other hand, is large and varies rapidly with frequency. The reflected waves may therefore combine at the input terminals in almost any fashion, and the effect they produce upon the input current will vary rapidly and violently as we proceed along the frequency scale. With given line impedance and voltage, however, the actual impedance of the terminal is related in simple fashion to the actual current entering it. The impedance, therefore, shows correspondingly wide fluctuations. The extremely irregular impedance and reflection coefficient characteristics of Fig. 2 exemplify the effect of reflections from the further junction points of the system. Another example is furnished by the curve of Fig. 7, which shows the reflection coefficient between the actual impedance of the filter of Fig. 6 when terminated in the line resistance, and that resistance. The humps of the curve come at frequencies whose phase shift is such that the wave reflected from the far end accentuates the departure of the near end image impedance from the desired value. The valleys correspond either to points at which the image impedances are ideal or to values of filter phase shift which cause the reflections at the two ends of the filter to correct one another.

Terminal Impedances Best Corrected by Special Type of Filter Section

Close impedance correction of these complicated characteristics seems hopeless. In order to keep the problem within manageable limits it is necessary to destroy the reflected waves at their source by preventing mismatches at all junctions between filters and other apparatus within the transmitting bands. The technical problem can consequently be reduced to the construction of a new type of filter section for use at terminations, the image impedance of the new section showing at one end a close approximation to the block type terminating impedance characteristic of Fig. 4-*A* or 4-*B* (i.e. a constant resistance) while at the other end it has the conventional rounded filter image impedances also shown on these figures, thus matching the standard sections forming the main bulk of the structure.

Early Improvements

Methods of approximating these characteristics to some extent were already available when the need for reducing crosstalk by impedance correction appeared. The first and longest step in this direction was

made by O. J. Zobel with his invention of "m-type" sections.[1] The schematic of a typical low-pass[2] filter terminated with these sections is shown in Fig. 6. The terminating networks are enclosed by the

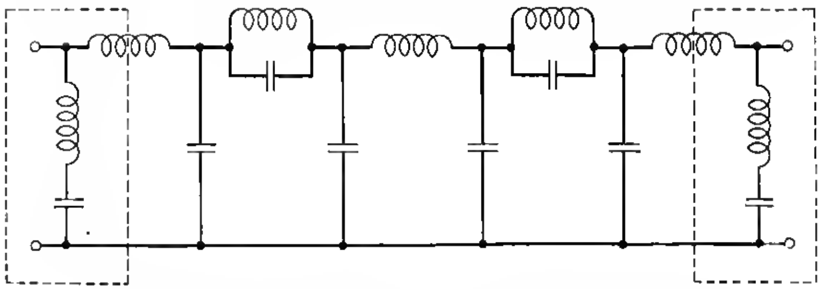


Fig. 6—A typical low-pass filter terminated in "*m*-derived" sections; $m = .512$. The reflection coefficient of this filter is given on Fig. 7.

broken lines. At one end these sections match the normal filter image impedance, as in Fig. 4-*A*. The approximation at the other end to the ideal block type characteristic of Fig. 4-*A* is shown by Fig. 4-*C*. The actual reflection coefficient of the filter of Fig. 6 is given on Fig. 7.

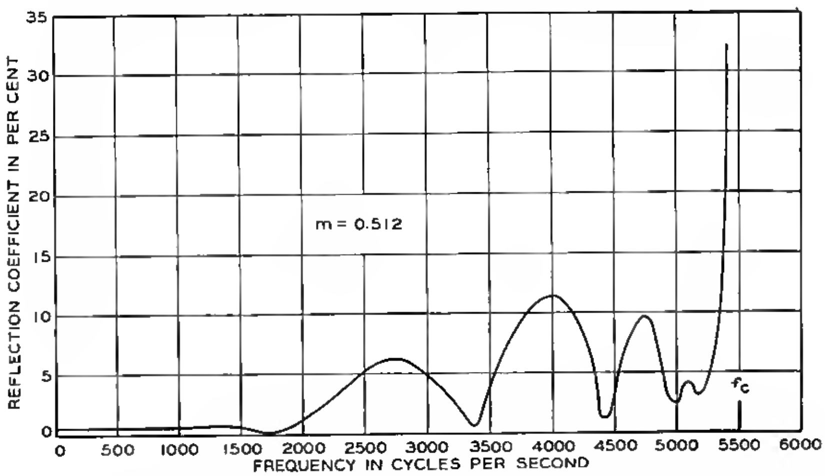


Fig. 7—Reflection coefficient of low-pass filter shown on Fig. 6.

[1] See *Bell System Technical Journal*, Jan. 1923.

[2] The m-type sections are applicable to all types of filters, low-pass, high-pass, or band-pass, and give very similar results in all cases. For example, the curve of Fig. 4-*D* can be considered as being a combination of two curves like that of Fig. 4-*C*, with a slight distortion of the frequency scale. If we allow for this distortion in scales the approximation to the ideal characteristic over a given percentage of the transmitted band is the same for low-pass and band-pass filters.

A modification of m-type sections, leading to the so called x-terminations,[3] is used when filters must be connected in parallel. The modification consists essentially in the elimination of the final shunt branches of the m-derived sections at the paralleling junction. Their places are taken in the transmitting band of either filter by the impedance of the

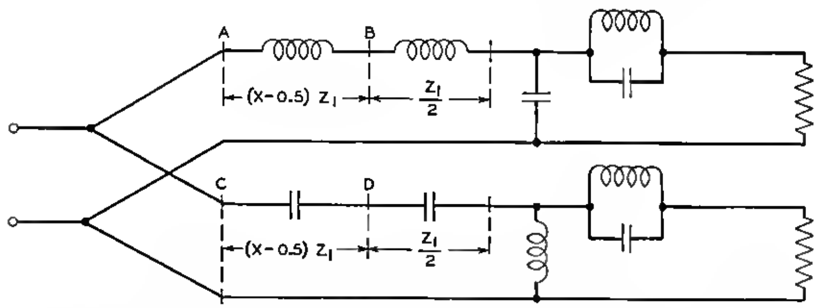


Fig. 8—Schematic of x-terminated filters—showing the way in which the parameter "x" determines the impedance which is added to each filter.

attenuating filter. A simple combination of low-pass and high-pass filters, having x-terminations at their common junction and m-type sections facing their load impedances is shown in Fig. 8. The terminating network for the low-pass filter consists of the impedance AB and that of the high-pass filter while the network for the high-pass filter

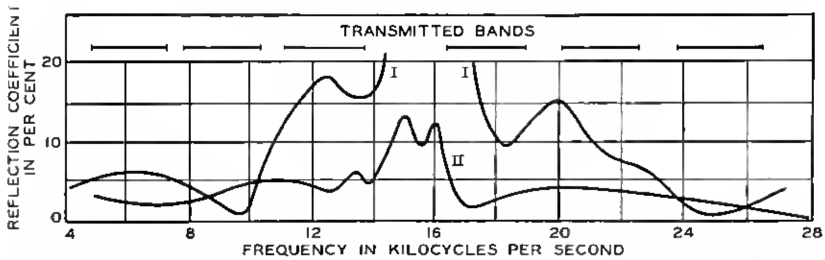


Fig. 9—Reflection coefficient characteristic of parallel low-pass and high-pass filters from the type "C" carrier telephone system.
I—Using x-terminations.
II—After the addition of a simple correcting network to the x-terminated filters.

is the impedance CD and that of the low-pass filter. The reflection coefficient of a typical pair of low-pass and high-pass filters from the Type C carrier telephone system, terminated similarly to the filters of Fig. 8, is shown by Curve I of Fig. 9.

These methods were supplemented by a number of more or less empirical schemes. For example, x-terminations, as Zobel described

[3] See U. S. Patent No. 1557230, issued to O. J. Zobel.

them, could be used only with complementary filters (i.e. low-pass and high-pass, or band-pass and band-elimination). In U. S. Patent No. 1616193 R. H. Mills specifically applies the method to band pass filters. Mills, proceeding from the fact that the adjacent sides of two

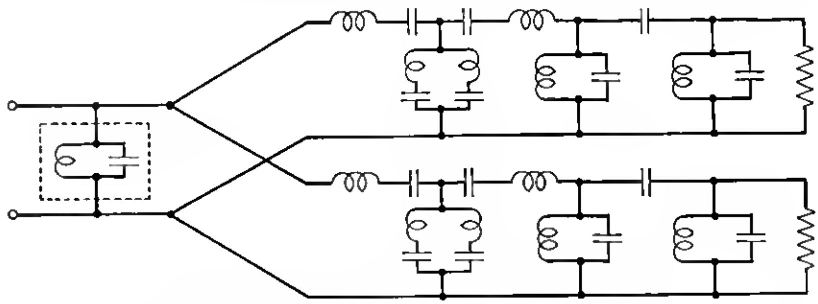


Fig. 10—Schematic of band-pass filters with auxiliary network. The reflection coefficient of these filters is given by curve I of Fig. 11.

band pass filters behave somewhat like complementary filters, while complementary filters are absent on the further sides of the bands, shunted simple networks, approximating the impedances of the missing complementary filters, across the parallel filter system. The filters of the Type *D* carrier telephone system[4] incorporated this device.

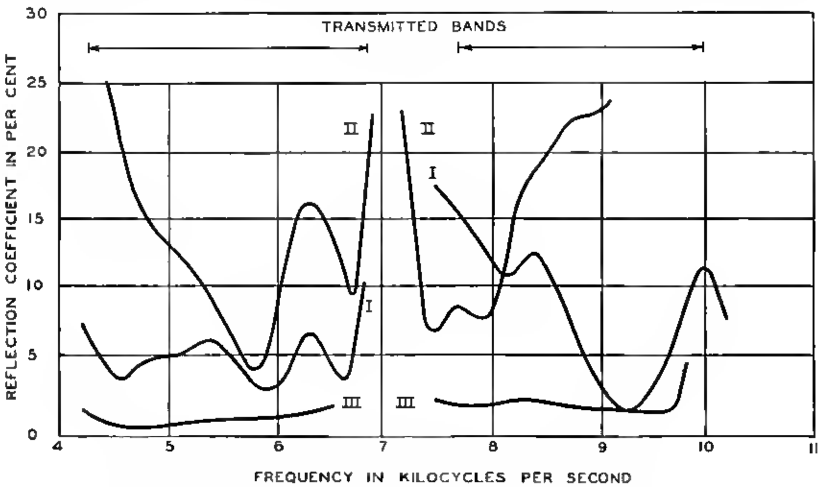


Fig. 11—Reflection coefficient of a set of band-pass filters.
I—For partially corrected x-terminated filters.
II—For uncorrected x-terminated filters.
III—For filters using the termination of Fig. 12-B modified for parallel operation.

[4] The general engineering features of this system are discussed in the *Transactions of the A. I. E. E.*, Vol. 48, No. 1, pp. 117–139.

The filter schematics are shown on Fig. 10, the auxiliary network being enclosed by broken lines. The performance of the filters was further improved by choosing terminating resistances differing somewhat from the nominal or mid-band, value of the filter image impedance. As a result of these two modifications the reflection coefficient characteristic shown by Curve I of Fig. 11 was obtained. Without them, the reflection coefficient would have been that given by Curve II.

The great improvement of filter impedance characteristics resulting from these devices is evident from a comparison of Figs. 7, 9 and 11 with Fig. 2. Instead of the reflection coefficients of 50 per cent or 60 per cent found in the earliest filters, the technique allows us to obtain reflection coefficients of the order of 10 per cent, within the frequency range of interest, for filters operating alone, of about 15 per cent for pairs of complementary filters in parallel, and of about 20 per cent for systems of parallel band-pass filters. These results were satisfactory for several years. The continued evolution of carrier systems toward higher and higher energy levels, however, and the constant increase in the number of systems in intimate physical association with one another, gradually made such standards inadequate. The reflection coefficient standards demanded by the severe crosstalk requirements of these systems have ranged from 2 per cent to 10 per cent in recent filter designs. It became evident some years ago that if these stringent reflection coefficient requirements were to be met a new analytical technique, more general and more powerful than its predecessors, would be necessary.

A New Technique and the Results of its Application to Impedance Correction

Such a technique has been developed. The method is essentially a generalization of the processes by which Zobel's "x-terminated" filters were derived. It leads to a series of filter sections, the number of which can be extended as far as is necessary to secure a satisfactory approximation to the desired image impedance characteristic.[5] The generalized configurations of several sections are given on Fig. 12. The a's and k's of this figure are design parameters, Z_{1k} and Z_{2k} refer to the filter with which the section is to be used. By choosing Z_{1k} and Z_{2k} appropriately the terminations can be adapted to any type of filter structure, whether low-pass, high-pass or band-pass.

The simplest of these sections can be shown to be equivalent to an "m-type" structure, and will naturally give the same results. A

[5] For a detailed discussion of the theory underlying this technique see, "A Method of Impedance Correction," appearing simultaneously in this journal.

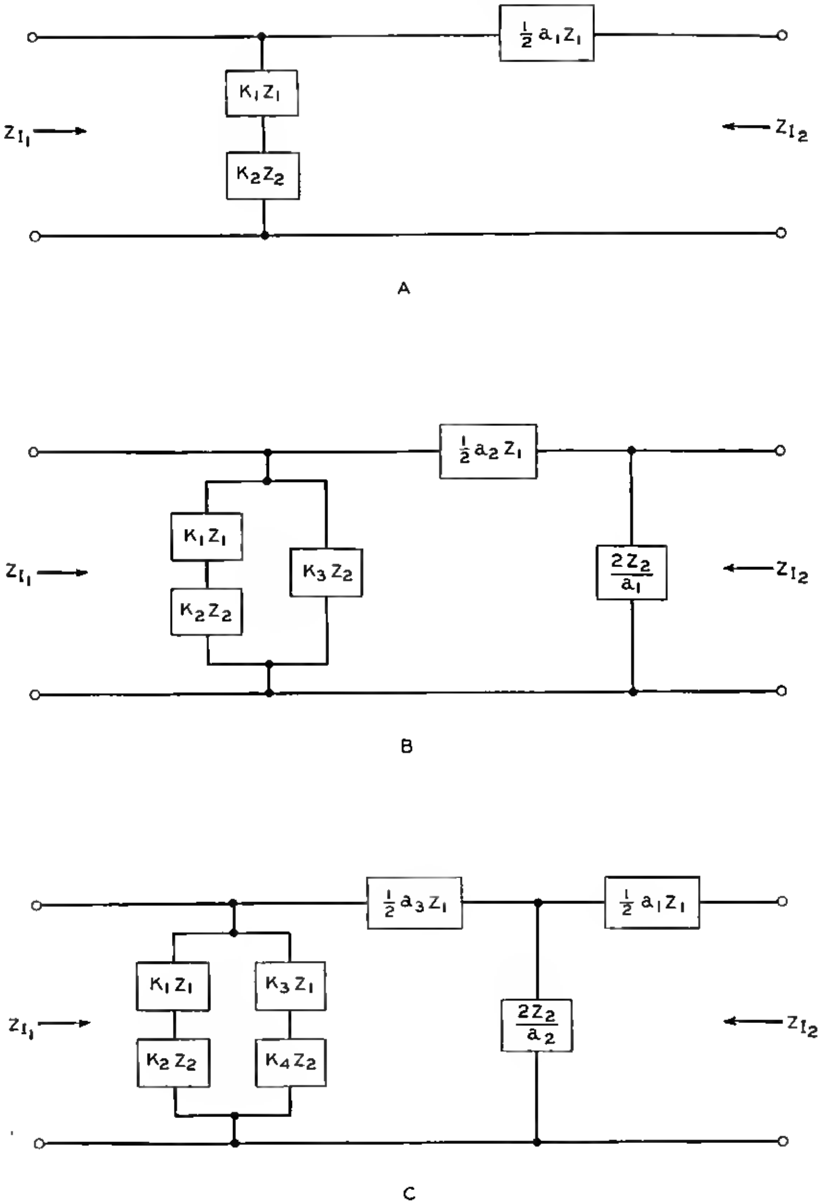


Fig. 12—Generalized schematics of terminating sections.

A—An "*m*-derived" or "single-branch" network.
B—A "2-branch" network.
C—A "3-branch" network.

typical image impedance[6] characteristic of the next more complicated structure (Fig. 12-*B*) is shown on Fig. 13. The vertical scale of this figure has been made considerably larger than that of Fig. 4 in order

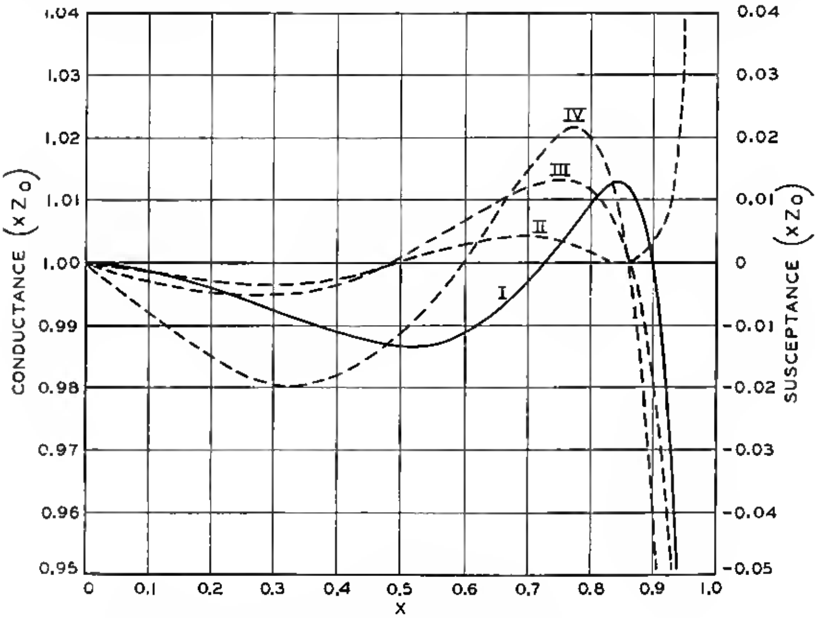


Fig. 13—Typical image impedance characteristic of a 2-branch termination (Fig. 12-*B*).
I—Real component.
II, III and IV—Various possible imaginary components.

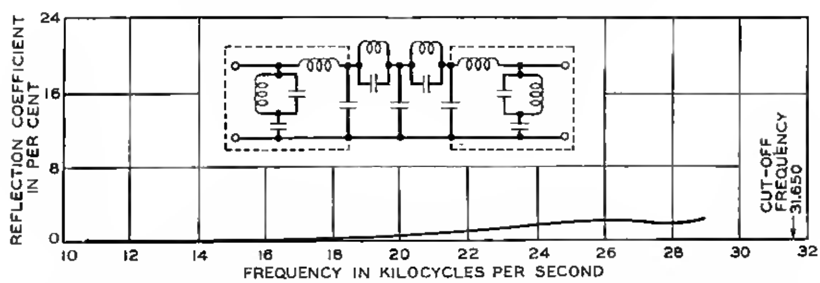


Fig. 14—Schematic and reflection coefficient of a low-pass filter with 2-branch terminations.

[6] The fact that the so-called "image impedances" of Figs. 12 and 16 contain slight imaginary components, in defiance of the fact that the image impedance of a reactive network is never a complex quantity, is traceable to the method used in analyzing the structures, which will be more fully understood from the discussion in the accompanying paper. The curves actually represent the impedance of the section when they are terminated in the filter image impedance. The reason for showing several curves for the imaginary component will be brought out later.

to bring out the departure of the image impedance from its ideal value more clearly. A low pass filter to which the termination has been applied is shown on Fig. 14. The terminating sections are enclosed by the broken lines. The resulting reflection coefficient is given on the same figure. It will be observed that the reflection coefficient over 93 per cent of the transmitting band is less than 2.5 per cent, or only about one-fourth that of the analogous m-type terminated filter shown in Fig. 6. The application of the structure to band-pass filters is illustrated in Fig. 15, which represents a portion of the redesigned type

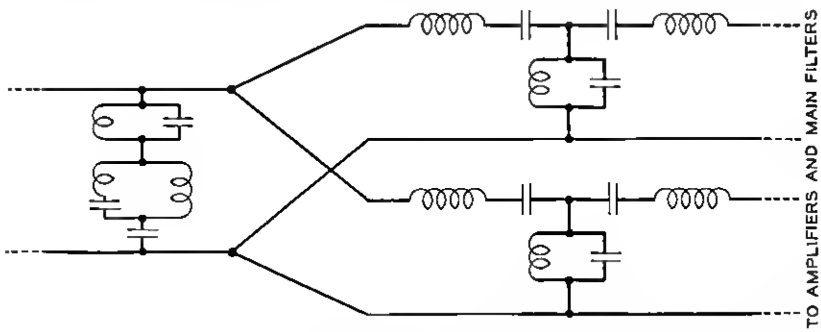


Fig. 15—Band-pass filters with 2-branch terminations modified for parallel operation. The reflection coefficient of these filters is given by Curve III, Fig. 11.

"*D*" system. The configuration shown in Fig. 12-*B* has been modified in these networks to adapt the filters for parallel operation. There are no separate terminations at the receiving ends of the filters since with these very simple structures reflection at the receiving ends could be taken into account by a slight adjustment of the terminating network facing the line. The improvement produced by the new networks in the reflection coefficient characteristics of the system is evident from a comparison of Curves III and I of Fig. 11.

The terminating section shown in Fig. 12-*B* which is one step beyond the "m-type" section in complexity, is adequate in most situations. When a severe reflection coefficient requirement must be met almost up to the filter cutoff, however, it is necessary to resort to the more complicated configuration of Fig. 12-*C*. The image impedance characteristic of this section, when its parameters are adjusted for an operating range extending over 97.5 per cent of the theoretical transmitting band, is shown on Fig. 16. The reflection coefficient actually obtained when sections of this type, but with somewhat different values of the design parameters, were applied to a high-pass filter is shown by Fig. 17. The filter configuration is given on the same figure. In this figure the

susceptance controlling network of the high-pass filter at the paralleled end is composed of the coil and condenser in series across the input terminals and the susceptance of the low-pass filter; at the other end

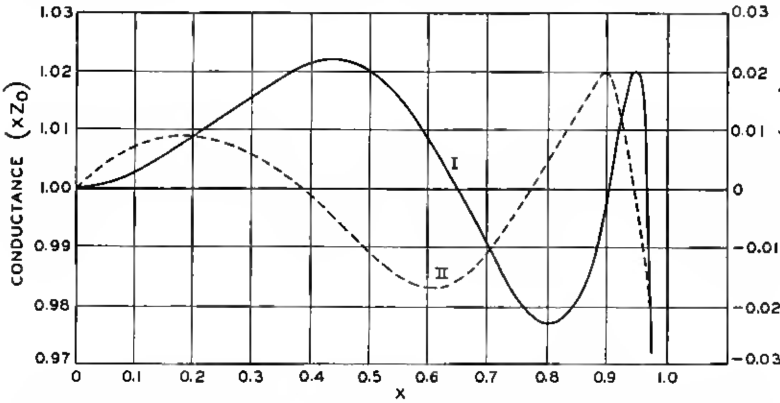


Fig. 16—Typical image impedance characteristic of a 3-branch termination (Fig. 12-*C*).
I—Real Component.
II—Imaginary component.

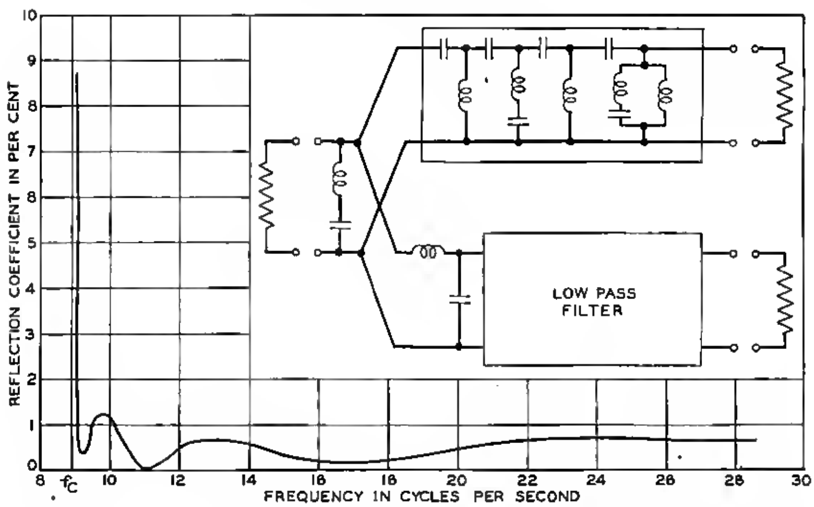


Fig. 17—Schematic and reflection coefficient of a high-pass filter with 3-branch terminations.

of the high-pass filter the three-element two-terminal network controls the susceptance. The conductance controlling sections at either end are composed of the first series condenser and the first shunt coil and a

portion of the second series condenser. The maximum reflection coefficient over about 95 per cent of the nominal transmitting band is slightly greater than 1 per cent. We can summarize these quantitative results in the rough statement that each of the three stages in the progress from the most primitive filter section to the relatively complicated network of Fig. 12-*C* appears to reduce the reflection coefficient obtainable over a given frequency range by a factor of about three or four.

Impedance Correction for Filters Operating in Parallel

The modifications which must be made in these sections in order to adapt them for use with filters which must operate in parallel are similar to those which were made in adapting "m-type" sections to this service. The final branch of each termination is omitted, its place being taken within the transmitting band of the filter to which it belongs, by the impedance of the parallel, attenuating, filters of the system. The parallel filters cannot however be relied upon to simulate the missing branch, even in this frequency range, with great accuracy. If we wish to preserve the high standards achieved by the terminations in other circumstances, therefore, it is, in general, necessary to introduce an auxiliary network in shunt with the circuit as a whole to improve the approximations to the missing branches. When this is done the reflection coefficient of the complete system is substantially identical in any transmitting range with that which would be obtained from the corresponding filter operating alone.

The thorough exploitation of the possibilities of these auxiliary networks leads to a marked improvement in the performance even of the well-known x-terminations. The reflection coefficient characteristic of a typical pair of high- and low-pass filters has already been shown by Curve I of Fig. 9. The high value of the reflection coefficient of these filters is largely due to the fact that neither filter in its attenuating range supplies quite enough admittance to take the place of the missing shunt branch of the other filter. The addition of a simple tuned circuit resonating between the transmitting bands to compensate for this deficiency in admittance reduces the reflection coefficient to the level shown by Curve II. The results for x-terminated band-pass filters are even more striking. Fig. 18 gives the susceptance at the line terminals of a set of three filters for several different conditions. The susceptance should ideally be zero. Curve I gives its value when no auxiliary network is added, Curve II, the level to which it is reduced by the auxiliary network suggested by Mills, and Curve III the characteristic which can be obtained with the help of a more elaborate auxiliary network. These curves can be given quantitative significance if

we notice that the deviation represented by Curve I at the point marked by the arrow would lead to a reflection coefficient of about 50 per cent even if the system were otherwise ideally terminated. Curves II and III, under the same conditions, represent reflection coefficients of about 29 per cent and 3 per cent respectively.

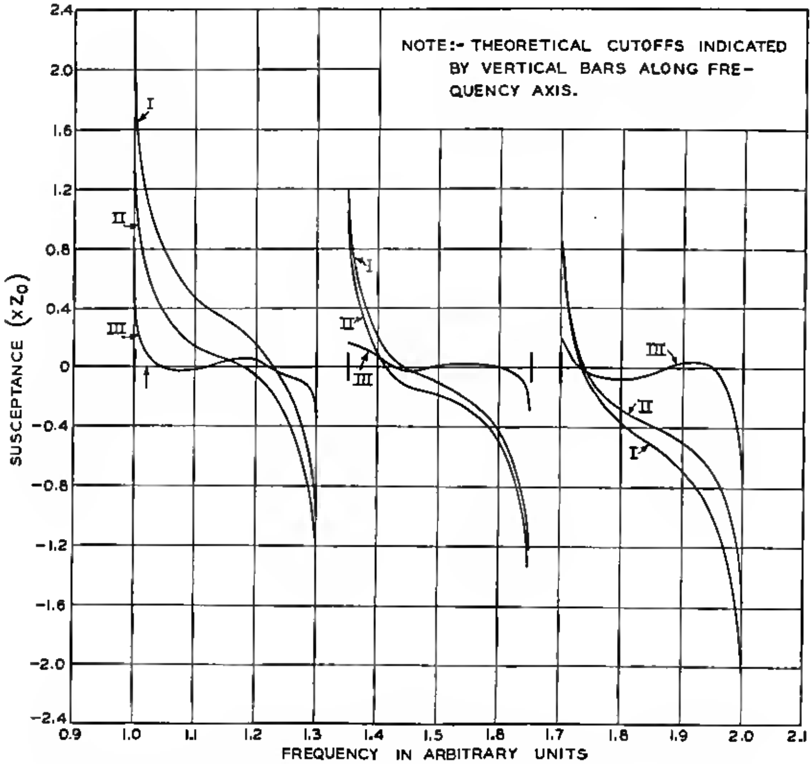


Fig. 18—Susceptance correction of x-terminated band-pass filters.
I—Uncorrected Susceptance.
II—Susceptance after the addition of a simple auxiliary network.
III—Susceptance after the addition of a more elaborate auxiliary network.

Improvements in Filters for Use with Modulator and Demodulator

We have hitherto restricted our attention to the impedance characteristics of filters within their transmitting bands since it is only in this range that impedance irregularities in the circuit can produce crosstalk. When a filter operates in conjunction with a modulating device, however, a high modulator efficiency with low distortion demands that the impedance of the filter to the untransmitted side band be low (or high) and nearly constant. All of the correcting networks

we have thus far described produce sharp changes in reactance of the attenuating region and are therefore unsuitable for such circuits. In spite of their poor characteristics within the transmitting band, therefore, it has hitherto been necessary to use mid-shunt image impedance terminations of the primitive "constant-k" type at junctions between filters and modulators. Curve I of Fig. 19 for example, shows the

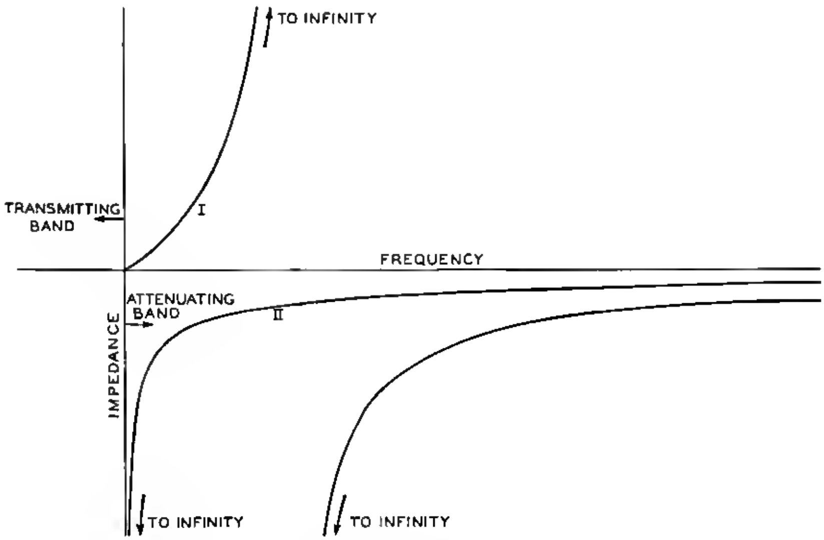


Fig. 19—"Constant-k" and "m-derived" type image impedances in the attenuating range.
I—"m-derived" type image impedance.
II—"Constant-k" type image impedance.

impedance characteristic of an "m-type" section beyond the cutoff in comparison with Curve II, representing the impedance of the "constant-k" type section in this range. The network configurations shown in Fig. 12 however represent only one of two possible classes of sections which can be developed as a result of the general analysis given in the accompanying paper. The other class is radically different in configuration. Networks of this second class may be advantageously substituted for the "constant-k" sections formerly used with modulators. The network characteristics in their attenuating regions approximate those of the "constant-k" sections and while they are not quite as good in the transmitting region as the characteristics furnished by the networks of Fig. 12 they are much better than the characteristics of the "constant k" sections of Figs. 4-*A* and 4-*B*.

Attenuation of Impedance Correcting Sections Reduces Net Cost of Impedance Correction

The economic aspects of filter design demand some sort of an evaluation of the cost of improving filter impedances. While the terminat-

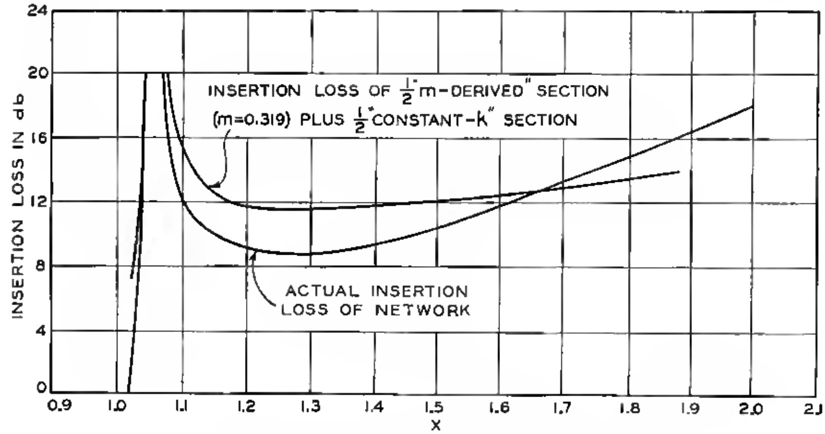


Fig. 20—Insertion loss of a 2-branch termination.

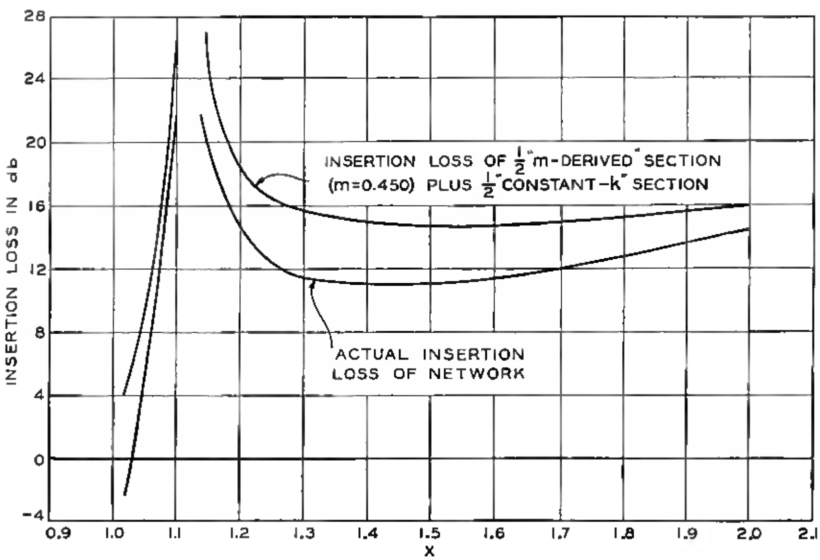


Fig. 21—Insertion loss of a 2-branch termination.

ing sections shown in Fig. 12 are rather complicated, their cost is discounted considerably by the fact that they contribute appreciably to the attenuation of the structure as a whole to undesired frequencies.

Moreover, their attenuation characteristics can be varied within fairly wide limits without appreciably affecting the impedance characteristics we obtain. If we make allowance beforehand for the attenuation of the terminations, therefore, the number of sections making up the main body of the filter can be correspondingly reduced. These relations are illustrated by Figs. 20, 21 and 22 which are drawn for terminations having the configuration of Fig. 12-*B*. The network attenuation is compared in each case with the attenuation of the most nearly equiv-

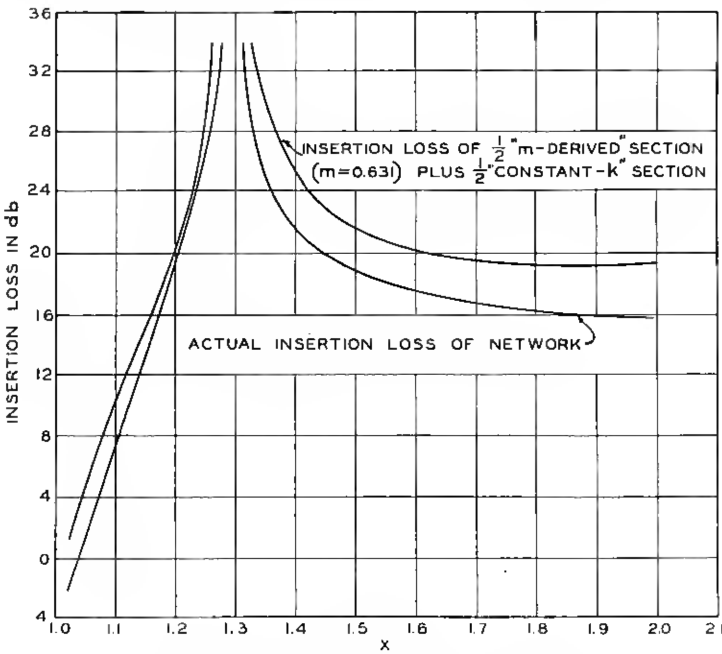


Fig. 22—Insertion loss of a 2-branch termination.

alent filter structure. The corresponding modifications in the impedance characteristic of the network are shown by Curves II, III and IV of Fig. 13. The real component (Curve I) of the impedance is the same in all cases, since the adjustment of the attenuation characteristic was produced entirely by manipulating the final series branch of the network, which has no effect on this component. When low- and high-pass filters are involved the terminating networks contain one more element than the suggested filter equivalent. This much must be conceded to the cost of impedance correction. It will be observed, however, that the remaining elements contribute almost as much attenuation as they would in standard filter sections. Indeed at fre-

quencies remote from the cutoff the attenuation of the network considerably exceeds that of the filter equivalent. The attenuation produced by the auxiliary (susceptance) networks used in conjunction with parallel filters is not so easily evaluated in terms of a standard filter equivalent. Since these networks produce peaks of attenuation just beyond the filter cutoff, thus enhancing the selectivity of the systems, they are however, in some respects particularly valuable. We can summarize the economic aspects of impedance correction in the statements that a severe impedance requirement will increase the number of elements (coils and condensers) required for an average filter used in carrier circuits by about 15 per cent or 20 per cent, and that the corresponding increase in the cost of the filter as a whole will be about 10 per cent or 15 per cent.

Practical Limitation to Impedance Correction

The fundamental limitation on the correction of wave filter impedances is practical rather than analytical. In other words it depends upon the accuracy with which it is possible to manufacture filters. All the curves so far exhibited have been based on the assumption that the filter elements, coils, condensers, and resistances, have the

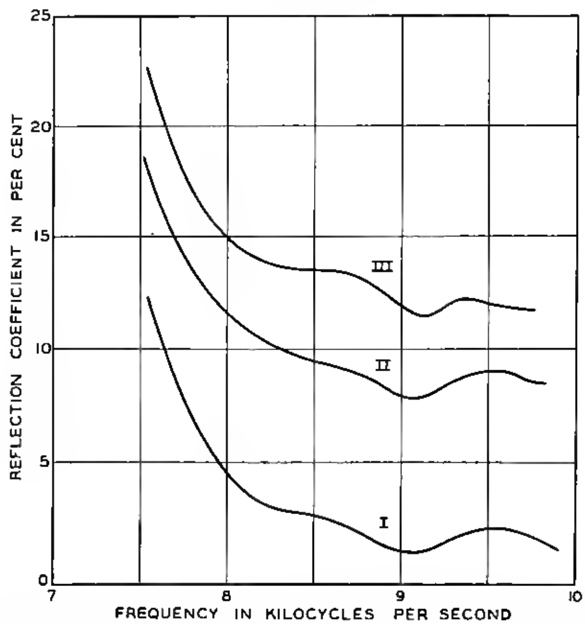


Fig. 23—Effect of element variations on a filter reflection coefficient.
I—Reflection coefficient when all elements have their exact design values.
II—Envelope of reflection coefficients of the best 99 per cent of the filters.
III—Envelope of worst possible reflection coefficients.

exact values ascribed to them by the design formulæ. The limitations of manufacture, however, demand that elements be permitted to deviate from their mean values by as much as 1 per cent or 2 per cent. These element deviations in general degrade the performance of filters by increasing their reflection coefficients. The increase in reflection coefficient is largely independent of the initial reflection coefficient, that is, it is about the same for a filter whose normal reflection coefficient is very small as for one of inferior theoretical design having a large normal reflection coefficient. Curve I of Fig. 23 shows the reflection coefficient of a particular filter when all of the elements have their design values; Curve III, the envelop of maximum reflection coefficients for this filter

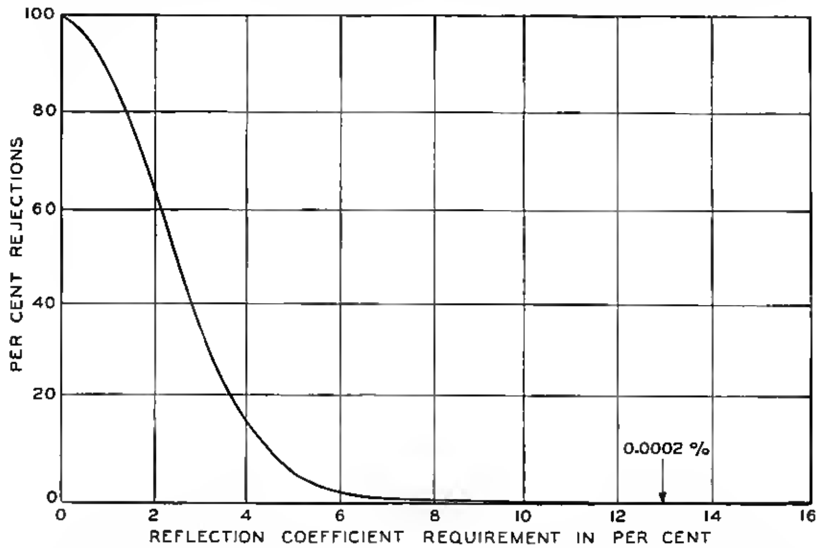


Fig. 24—Distribution of the reflection coefficients of a given filter at one frequency in terms of the percentage rejected in meeting any requirement.

when the coils are permitted to vary $\pm$ 1.5 per cent and the condensers $\pm$ 0.8 per cent from their design values; Curve II the maximum reflection coefficient for the best 99 per cent of the filters manufactured to these limits. The last curve is based on a priori probability computation which assumes that the distribution curves of the element deviations follow the normal law. A distribution curve of reflection coefficients for another filter at a particular frequency plotted in terms of the per cent of filters that would be rejected in meeting any requirements is given in Fig. 24. Studies of this sort are too extensive to be included in this paper but they have shown that the advances in the technique of impedance correction here recorded are well ahead of the practical limitations of the problem.